

Axial Vector Coupling Constant in Chiral Colour Dielectric Model

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Abstract

The axial vector coupling constants of the β decay processes of neutron and hyperon are calculated in SU(3) chiral colour dielectric model (CCDM). Using these axial coupling constants of neutron and hyperon, in CCDM we calculate the integrals of the spin dependent structure functions for proton and neutron. Our result is similar to the results obtained by MIT bag and Cloudy bag models.

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1 Introduction

In terms of quark parton model (QPM) interpretation the EMC[1] result leads to a negligible contribution of the quark spins to the proton spin. This is in clear contradiction with the explanation given by the naive quark picture, in which the proton spin comes from the sum of the quark spins. Moreover it is argued that the strange quark sea is polarized opposite to that of the proton[2, 3]. So the spin carried by u and d quarks cancel with that of the strange quark. Thus the total helicity carried by the quarks is very small. The EMC experiment has been complimented by measurements of the neutron spin structure function in the E142 experiment at SLAC and by the SMC deuteron experiment[4]. All these combined datas are thoroughly analysed by different groups[2, 3].

The Bjorken-sum[5] rule relates the polarized structure function for proton and neutron with the axial vector coupling constant g_A and the vector coupling constant g_V of the nucleon and is given (without QCD correction) as

$$\int_0^1 (g_1^p(x) - g_1^n(x)) dx = \frac{1}{6} \left| \frac{g_A}{g_V} \right|. \quad (1)$$

Similarly the Ellis-Jaffe (EJ)[6] sum rule for proton and neutron are given as

$$\int_0^1 g_1^{p(n)}(x) dx = \frac{1}{12} \frac{g_A}{g_V} \left(+ (-)1 + \frac{5}{3} \frac{F/D - 1}{F/D + 1} \right), \quad (2)$$

where F and D are antisymmetric and symmetric weak $SU(3)$ couplings measurable in the beta decay of the neutron and hyperons in baryon octet. This is derived with the assumptions that $SU(3)$ flavour symmetry of the baryon octet is exact and strange quarks in the nucleon are unpolarized. In the above equations the vector and axial vector coupling constants are calculated from the low-momentum transfer limit. But the polarized structure functions are calculated from the high momentum transfer limit. So basically the above sum rules relate both the low and high momentum transfer phenomena. By using the current algebra, one can show that the integral of the nucleon structure function can be given by the matrix element

$$\int_0^1 g_1^N(x) dx = \frac{1}{2} \langle N \uparrow | \bar{\psi}(0) Q^2 \gamma_z \gamma_5 \psi(0) | N \uparrow \rangle, \quad (3)$$

where $g_1^N(x)$ in terms of the spin dependent quark distribution function is given as

$$g_1^N(x) = \frac{1}{2} \sum_i Q_i^2 [q_f^\uparrow(x) - q_f^\downarrow(x)]. \quad (4)$$

Here $q_f^{\uparrow(\downarrow)}(x)$ is the number density for quarks of flavour f and charge Q_f at momentum fraction x , and with spin parallel (anti-parallel) to that of the nucleon. So we can calculate the axial coupling constants for neutron and hyperon beta decay processes from any phenomenological model of hadron and relate that to the structure functions. This is the basis of our calculation in this paper. We have used the $SU(3)$ chiral colour dielectric model (CCDM)[7] to calculate the axial coupling

constants for the process $n \rightarrow pe^- \bar{\nu}_e$ and the hyperon beta decay ($\Sigma^- \rightarrow ne^- \bar{\nu}_e$) process and use the above sum rules to predict the integral of the spin structure functions of the nucleon.

The chiral symmetry has long been known to be an important symmetry of the strong interaction. Massless QCD is invariant under this symmetry. In the context of σ -model, Gell-Mann and Levy[8] showed that nucleon acquires mass via the spontaneous breaking of chiral symmetry. Later on it was realised that this symmetry has to be incorporated in all the phenomenological models of hadron. In MIT bag model[9], the chiral symmetry is violated due to the fact that, the reflected quark from the bag surface does not flip its spin, hence the quark with helicity +1 after reflection will flip its helicity to -1 , thus violating the chirality. On the other hand, the bag model with pion (Cloudy bag model) conserve the chiral symmetry[9]. The change in the helicity due to reflection is compensated by the emission of a p -wave pion of unit orbital angular momentum leaving the bare three quark system (this system might be nucleon or delta). So the quarks spin inside the baryon may not add up to $\pm \frac{1}{2}$. So the spin structure of the nucleon revealed by the electromagnetic probe might be different. The chiral SU(2) version of Cloudy bag model[9], Friedberg-Lee soliton model[10] and Colour Dielectric model have been studied extensively[11]. All these calculations yield a good agreement with the static hadronic properties. These models have been extended to include SU(3) chiral symmetry to study the pseudo scalar meson octet contribution[7, 12] although this symmetry is badly broken due to the different masses of the pion, kaon and eta mesons.

2 Model

Unlike other models, the colour dielectric model generates the absolute confinement dynamically. The scalar field in the model lagrangian takes into account the long range order effect due to non-perturbative QCD vacuum and the short wave-length components do not exist. As the scalar field takes into account the long distance behaviour, the gluonic fields are treated perturbatively. For $\chi \rightarrow 0$, the inverse coupling of scalar field to quarks i.e. $\frac{m_q}{\chi} \bar{\psi} \psi$ dynamically confines the quarks; even in the absence of gauge field. The chiral SU(2) version of CDM has been successfully used to study the nucleon static properties. The SU(3) chiral colour dielectric model has also been used to study the strangeness related phenomena.

The SU(3) CCDDM lagrangian is given by[7]

$$\begin{aligned} \mathcal{L} = & \sum_i \bar{\psi}_i \left[i\gamma^\mu \partial_\mu - \left(m_{su} + \frac{m_u}{\chi} \left(1 + \frac{i}{f_\phi} \gamma_5 \lambda \cdot \phi \right) \right) - \frac{1}{2} g_s \gamma^\mu \lambda_a A_\mu^a \right] \psi_i \\ & - \frac{1}{4} \kappa(\chi) F_{\mu\nu}^a F^{a,\mu\nu} + \frac{1}{2} \sigma_v^2 (\partial_\mu \chi)^2 + \frac{1}{2} (\partial_\mu \vec{\phi})^2 - \frac{1}{2} m_\phi^2 \phi^2 - U(\chi), \end{aligned} \quad (5)$$

where ψ , χ , ϕ and A_μ^a are effective quark, colour dielectric, SU(3) pseudo scalar meson and the gluon fields respectively. m_ϕ is the octet meson mass, f_ϕ is the meson decay constant, $\alpha_s = g_s^2/4\pi$ is the strong coupling constant and $F^{a,\mu\nu}$ is the colour electromagnetic field tensor. The sum i

is over quark colour and flavour and the effective quark mass is $(m_{su} + \frac{m_u}{\chi})$ with $m_{su} = 0$ for u and d quarks. The gluon field interacts with the dielectric field through a dielectric functional $\kappa(\chi) = \chi^4(x)$ and the self-interaction of the dielectric field is given as

$$U(\chi) = B[\alpha\chi^2 - 2(\alpha - 2)\chi^3 + (\alpha - 3)\chi^4]. \quad (6)$$

For $\alpha > 6$, $U(\chi)$ has a double well structure, with an absolute minimum at $\chi=0$ and local minimum at $\chi=1$ and the energy density difference between these two minima gives the bag constant B . The mass of the scalar field is given as $m_{GB} = \sqrt{\frac{2B\alpha}{\sigma_v^2}}$ and this is interpreted as the glueball mass. The strong coupling constant is calculated by fitting the nucleon and the delta masses.

In SU(3) CCDDM picture the physical baryon is a system of three quarks surrounded by a meson cloud. Following the approach of Thomas et al.[9], the physical baryon state $|A\rangle$ can be expressed as

$$|A\rangle = \sqrt{P_A} \left\{ 1 + (m_A - \tilde{H}_0)^{-1} H_{int} \right\} |A_0\rangle. \quad (7)$$

Here P_A is the probability of finding bare baryon (three quarks) state $|A_0\rangle$ with bare mass m_{A0} and m_A is the physical baryon mass. H_0 is the noninteracting hamiltonian which includes quark and the dielectric field hamiltonian and free meson part. H_{int} is the quark- meson interaction hamiltonian. The second term in the above equation is responsible for the generation of meson cloud around the bare baryon. In this picture the meson field is considered to be small, so that non- linearities due to meson coupling can be neglected and meson contribution can be treated perturbatively.

3 Semileptonic Decay

The β decay process between octet baryons A and B is $A \rightarrow Be^- \bar{\nu}_e$. For the calculation of the axial coupling constant (semileptonic decay constant) we have to evaluate the matrix element of the axial vector current (isospin changing) $\langle B | A_\mu(x) | A \rangle$, where $|A\rangle$ and $|B\rangle$ are initial and final baryon states, and the axial vector current is

$$\begin{aligned} A_\mu(x) &= \bar{\psi}(x) \gamma_\mu \gamma_5 \frac{\lambda}{2} \psi(x) \\ &= \bar{\psi}(x) \gamma_\mu \gamma_5 \frac{1}{2} (\lambda_1 + i\lambda_2) \psi(x) + \bar{\psi}(x) \gamma_\mu \gamma_5 \frac{1}{2} (\lambda_4 + i\lambda_5) \psi(x) \\ &= A_{\mu 1}(x) + A_{\mu 2}(x). \end{aligned} \quad (8)$$

In the Eq.(8) the first term of the RHS correspond to strangeness change 0 ($\Delta S = 0$) and the second term is for strangeness change 1 ($\Delta S = 1$).

Since the quark-meson coupling is linear in the meson field and the mesonic part of the axial current is proportional to the divergence of the the meson field, there will be no contribution to semileptonic decay constant from the meson cloud[14]. Thus the contribution to the semileptonic

decay constant comes from the quark part of the axial current only. The quark part of the axial coupling constant is calculated from the matrix element[7, 9] $\langle B | A_\mu(x) | A \rangle$ and is given as

$$\begin{aligned} g_A &= \langle B | A_z(x) | A \rangle \\ &= \sqrt{P_A P_B} \langle B_0 | A_z(x) | A_0 \rangle \\ &\quad + \sqrt{P_A P_B} \langle B_0 | H_{int}(m_B - \tilde{H}_0)^{-1} A_z(x) (m_A - \tilde{H}_0)^{-1} H_{int} | A_0 \rangle. \end{aligned} \quad (9)$$

Summing over the intermediate states and after some algebra, the second term in the above equation can be expressed in terms of $3j$ symbols and can be written as

$$\begin{aligned} &\sqrt{P_A P_B} \langle B_0 | H_{int}(m_B - \tilde{H}_0)^{-1} A_z(x) (m_A - \tilde{H}_0)^{-1} H_{int} | A_0 \rangle \\ &= \frac{\sqrt{P_A P_B}}{3\pi} \sum_{C,D} \left(\frac{f_{BC\phi} f_{AD\phi}}{m_\phi^2} \right) 2\eta \sqrt{(2T_A + 1)(2T_B + 1)} \\ &\quad \times \sum_{m_1, m_2, i} \begin{pmatrix} \frac{1}{2} & 1 & S_C \\ -s_B & m_1 & s_C \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 1 & S_D \\ -s_A & m_2 & s_D \end{pmatrix} \\ &\quad \times \begin{pmatrix} T_B & T & T_C \\ -t_B & i & t_C \end{pmatrix} \begin{pmatrix} T_A & T & T_D \\ -t_A & i & t_D \end{pmatrix} \\ &\quad \times \int dk \frac{k^4 u_{BC}(k) u_{AD}(k) \langle C_0 | A_z | D_0 \rangle}{(\omega_{CB} + \omega_k)(\omega_{DA} + \omega_k)\omega_k} \\ &= \sqrt{P_A P_B} \sum_{C,D} g(C, D), \end{aligned} \quad (10)$$

where $f_{BC\phi}$ and $f_{AD\phi}$ are the baryon-meson coupling constants, $u(k)$ is the baryon-meson form factor and $\omega_k = \sqrt{(k^2 + m_\phi^2)}$. The phase factor is

$$\eta = (-1)^{(T_A + T_B - s_A - s_B - t_A - t_B + 1)}. \quad (11)$$

Thus the axial coupling constant is given as

$$g_A = \sqrt{P_A P_B} \left[\langle B_0 | A_z(x) | A_0 \rangle + \sum_{C,D} g(C, D) \right]. \quad (12)$$

The matrix elements $\langle B_0 | A_z(x) | A_0 \rangle$ and $\langle C_0 | A_z(x) | D_0 \rangle$ can be evaluated using the wave functions of the respective baryons. The second term in Eq.(12) corresponds to the meson exchange contribution.

4 Results

Briefly, the calculation in perturbative CCDDM proceeds as follows. One first solves the quark and dielectric field equations in mean field approximation and constructs the (bare) baryon states consisting of three quarks. In our calculation Peierls-Yoccoz momentum projection technique[11,

13] is used to construct good momentum states and these states are used to calculate the bare baryon properties and baryon-meson form factors. The pseudo scalar meson-quark interaction is then included to calculate the mesonic effects on baryon properties.

The parameters in the CCDM are m_{GB} , m_u , m_s , α_s , B , f_ϕ and α . Our calculations show that the parameter α , which determines the height of the maximum of $U(\chi)$ between two minima at $\chi = 0$ and $\chi = 1$ does not play an important role in the calculation. We have chosen $\alpha = 36$ throughout. Also, the meson-quark coupling constant f_ϕ has been chosen to be 93 MeV, the pion decay constant. The rest of the parameters are varied to fit the properties of octet and decuplet baryons. We find that the masses of these baryons can be fitted, to a very good accuracy, for a family of parameter sets. The numerical results for our calculation for different parameter sets are shown in table 1. For all these cases, the baryon octet and decuplet masses are reproduced very well. There is, however, a large variation in the calculated static properties (charge radius, magnetic moment, axial coupling constant etc.). Generally small glueball masses give a reasonable agreement with the static properties (except for magnetic moments). As the glueball mass is increased, magnitudes of charge radii, magnetic moments and axial coupling constant decrease. This can be attributed to the increase in percentage of meson cloud with the increase in the glueball mass[7]. We have also shown the results obtained from different quark models[15]. We have shown results for four parameter sets. The four parameter sets are given as follows:

Set A: $m_{GB}=1050$ MeV, $m_u=105$ MeV, $m_s=318$ MeV and $B^{1/4}=94.5$ MeV

Set B: $m_{GB}=804$ MeV, $m_u=88$ MeV, $m_s=307$ MeV and $B^{1/4}=88.4$ MeV

Set C: $m_{GB}=4019$ MeV, $m_u=80$ MeV, $m_s=294$ MeV and $B^{1/4}=180$ MeV

Set D: $m_{GB}=1016$ MeV, $m_u=38$ MeV, $m_s=311$ MeV and $B^{1/4}=114$ MeV

The table 1 shows the g_A for neutron and Σ beta decays ($g_{A_{np}}$ and $g_{A_{\Sigma n}}$). The g_V of nucleon and hyperons are normalized to unity. We have used these two coupling constants to calculate the F/D ratio. Our calculation shows that the g_A for above two decay processes are less than the observed values. This is because the bare probabilities for nucleon (P_n) and sigma (P_Σ) are less than one. For example for the parameters of row one of the table, the probability of bare nucleon and sigma are respective 0.83 and 0.87. So the inclusion of meson reduces the g_A for neutron and Σ by 17% and 13% respectively. This reduction could have compensated by meson exchange contribution. But it is observed that the meson contribution is very small to over come the reduction. Also the kaon and eta contributions are opposite to that of the pion contribution. So this also reduces the axial coupling constant.

The F and D calculated from the above two baryon decays are shown in the table. It shows that for large glueball mass (Set C) these two SU(3) couplings are small compared to the one obtained from comparatively smaller glueball masses. But the ratio F/D is constant through out. We found the ratio $F/D \simeq 0.67$ over wide range variation of parameters. This ratio is higher than the value obtained by Close and Roberts[3] and Ehrnsperger et al.[16] The values for F and D obtained by

Close et al. are 0.459 ± 0.008 and 0.798 ± 0.008 ($F/D=0.575 \pm 0.016$) respectively and the value for F/D in ref[[16]] is 0.49 ± 0.08 .

The calculation of EJ sum rule for proton shows that our results agree with the old result[6] and very large compared to the EMC measurement[1]. Comparison with MIT bag model and CBM[15], shows that all the results are similar. We get small, non-zero and negative value for the neutron spin structure function, which is consistent with the experimental data. This non-zero contribution is solely attributed due to the meson exchange term. But this value is too small compared to the observed one. On the other hand neutron structure function is zero in non relativistic quark model (NRQM) and MIT bag model as shown in the table[15]. The NRQM gives a very large value of the g_A for neutron beta decay. Also it is seen that the proton structure function obtained using this coupling constant is very high.

Finally, our calculation shows that the proton spin structure function calculated using the axial coupling constant is very large compared to the EMC result. It agrees with the old result and also with MIT bag and CBM results. On the other hand the meson cloud in CCDDM and CBM give a non-zero and negative contribution to the neutron spin structure function, which is consistent with the analysis of the recent experimental datas. The above analysis of our results show that the CCDDM overestimate the value of the proton spin structure function. Infact it is true for all the quark models.

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Table 1: The results for four parameter sets of CCDDM are shown in the table. The NRQM, MIT bag and CBM results are also shown from ref[14]. The $g_{A_{np}}$ for CBM is normalized to the observed value at 0.8 fm of the proton charge radius. The experimental values for $g_{A_{np}}=1.254\pm.006$, $g_{A_{\Sigma n}}=0.340$, $\int g_1^p(x)dx=0.114\pm0.012\pm0.026$ and $\int g_1^n(x)dx=-0.077\pm0.012\pm0.026$. The value of $F=0.459\pm0.008$, $D=0.798\pm0.008$ and $F/D=0.575\pm.016$ in ref[4] and $F/D=0.49\pm0.08$ in ref[16].

	$g_{A_{np}}$	$g_{A_{\Sigma n}}$	$\int g_1^p(x)dx$	$\int g_1^n(x)dx$	F	D
A	1.141	0.229	0.190	-0.00032	0.456	0.685
B	1.166	0.234	0.194	-0.00026	0.466	0.700
C	0.908	0.185	0.150	-0.00095	0.361	0.546
D	1.180	0.237	0.197	-0.00018	0.472	0.709
NRQM	1.67		0.28	0		
MIT bag	1.09		0.18	0		
CBM	1.254		0.193	-0.010		

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